HUREWICZ THEOREM FOR EXTENSION DIMENSION

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Abstract. We prove a new selection theorem for multivalued mappings of C-space. Using this theorem we prove extension dimensional version of Hurewicz theorem for a closed mapping $f\colon X \to Y$ of k-space X onto paracompact C-space Y: if for finite CW-complex M we have e-dim$Y \leq [M]$ and for every point $y \in Y$ and every compactum Z with e-dim$Z \leq [M]$ we have e-dim$(f^{-1}(y) \times Z) \leq [L]$ for some CW-complex L, then e-dim$X \leq [L]$.

1. Introduction

The classical Hurewicz theorem states that for a mapping of finite-dimensional compacta $f\colon X \to Y$ we have

$$\dim X \leq \dim Y + \dim f, \quad \text{where} \quad \dim f = \max\{\dim(f^{-1}(y) \mid y \in Y\}.$$

There are several approaches to extension dimensional generalization of Hurewicz theorem [6],[3],[1],[7],[8],[9].

Using the idea from [3] we improve Theorem 7.6 from [1]:

Theorem 3.1. *Let $f\colon X \to Y$ be a closed mapping of a k-space X onto paracompact C-space Y. Suppose that* e-dim$Y \leq [M]$ *for a finite CW-complex M. If for every point $y \in Y$ and for every compactum Z with* e-dim$Z \leq [M]$ *we have* e-dim$(f^{-1}(y) \times Z) \leq [L]$ *for some CW-complex L, then* e-dim$X \leq [L]$.

The notion of extension dimension was introduced by Dranishnikov [4]: for a CW-complex L a space X is said to have *extension dimension* $\leq [L]$ (notation: e-dim$X \leq [L]$) if any mapping of its closed subspace $A \subset X$ into L admits an extension to the whole space X.

To prove Theorem 3.1 we need an extension dimensional version of Uspenskij's selection theorem [11]. In section 2 we prove Theorem 2.8 on selections of multivalued mappings of C-space. Then Theorem 2.5 helps us to prove Theorem 2.9 — a needed version of Uspenskij's theorem.

Filtrations of multivalued maps are proved to be very useful for construction of continuous selections [10], [1]. And we state our selection theorems in terms

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of filtrations. Note that Valov [12] used filtrations to prove a selection theorem for mappings of finite C-spaces.

Let us recall some definitions and introduce our notations. A space X is called a *k-space* if $U \subset X$ is open in X whenever $U \cap C$ is relatively open in C for every compact subset C of X. The *graph* of a multivalued mapping $F\colon X \to Y$ is the subset $\Gamma_F = \{(x,y) \in X \times Y : y \in F(x)\}$ of the product $X \times Y$.

We denote by $\mathrm{cov}X$ the collection of all coverings of the space X. For a cover ω of a space X and for a subset $A \subseteq X$ let $\mathrm{St}(A,\omega)$ denote the star of the set A with respect to ω. We say that a subset $A \subset X$ *refines* a cover $\omega \in \mathrm{cov}X$ if A is contained in some element of ω. A covering $\omega' \in \mathrm{cov}X$ *strongly star refines* a covering $\omega \in \mathrm{cov}X$ if for any element $W \in \omega'$ the set $\mathrm{St}(W,\omega')$ refines ω.

Definition 1.1. A topological space X is called *C-space* if for each sequence $\{\omega_i\}_{i\geq 1}$ of open covers of X, there is an open cover Σ of X of the form $\cup_{i=1}^{\infty}\sigma_i$ such that for each $i \geq 1$, σ_i is a pairwise disjoint collection which refines ω_i.

If the space X is paracompact, we can choose the cover Σ to be locally finite and every collection σ_i to be discrete.

Definition 1.2. A multivalued mapping $F\colon X \to Y$ is said to be *strongly lower semicontinuous* (briefly, strongly l.s.c.) if for any point $x \in X$ and any compact set $K \subset F(x)$ there exists a neighborhood V of x such that $K \subset F(z)$ for every $z \in V$.

Definition 1.3. Let L be a CW-complex. A pair of spaces $V \subset U$ is said to be *$[L]$-connected* (resp., *$[L]_c$-connected*) if for every paracompact space X (resp., compact metric space X) of extension dimension $\text{e-dim}X \leq [L]$ and for every closed subspace $A \subset X$ any mapping of A into V can be extended to a mapping of X into U.

An increasing[1] sequence of subspaces $Z_0 \subset Z_1 \subset \cdots \subset Z$ is called a *filtration* of space Z. A sequence of multivalued mappings $\{F_k\colon X \to Y\}$ is called a *filtration of multivalued mapping* $F\colon X \to Y$ if $\{F_k(x)\}$ is a filtration of $F(x)$ for any $x \in X$.

Definition 1.4. A filtration of multivalued mappings $\{G_i\colon X \to Y\}$ is said to be *fiberwise $[L]_c$-connected* if for any point $x \in X$ and any i the pair $G_i(x) \subset G_{i+1}(x)$ is $[L]_c$-connected.

2. Selection theorems

The following notion of stably $[L]$-connected filtration of multivalued mappings provides a key property of the filtration for our construction of continuous selections.

[1] We consider only increasing filtrations indexed by a segment of the integral series.

Definition 2.1. A pair $F \subset H$ of multivalued mappings from X to Y is called *stably* $[L]$*-connected* if every point $x \in X$ has a neighborhood O_x such that the pair $F(O_x) \subset \cap_{z \in O_x} H(z)$ is $[L]$-connected.

We say that the pair $F \subset H$ is called *stably* $[L]$*-connected with respect to a covering* $\omega \in \mathrm{cov}X$, if for any $W \in \omega$ the pair $F(W) \subset \cap_{x \in W} H(x)$ is $[L]$-connected.

A filtration $\{F_i\}$ of multivalued mappings is called *stably* $[L]$*-connected* if every pair $F_i \subset F_{i+1}$ is stably $[L]$-connected.

Clearly, any stably $[L]$-connected pair of multivalued maps of a space X is stably $[L]$-connected with respect to some covering of X.

We denote by Q the Hilbert cube. We identify a space Y with the subspace $Y \times \{0\}$ of the product $Y \times Q$ and denote by pr_Y the projection of $Y \times Q$ onto Y.

Definition 2.2. For a subspace $Z \subset Y \times Q$ we say that Y *projectively contains* Z. We say that a multivalued mapping $F \colon X \to Y$ *projectively contains* a multivalued mapping $G \colon X \to Y \times Q$ if for any point $x \in X$ the set $\mathrm{pr}_Y \circ G(x)$ is contained in $F(x)$.

Lemma 2.3. *Let L be a finite CW-complex. If a topological space Y contains a compactum K of extension dimension* $\text{e-dim}K \le [L]$ *such that the pair $K \subset Y$ is $[L]_c$-connected, then Y projectively contains a compactum K' of extension dimension* $\text{e-dim}K' \le [L]$ *such that K lies in K' and the pair $K \subset K'$ is $[L]$-connected.*

Proof. There exists $AE([L])$-compactum K' of extension dimension $\text{e-dim}K' \le [L]$ containing the given compactum K [2]. Clearly, the pair $K \subset K'$ is $[L]$-connected. Since $\text{e-dim}K' \le [L]$, there exists a mapping $p \colon K' \to Y$ extending the inclusion of K into Y.

It is easy to see that there exists a mapping $q \colon K' \to Q$ such that $q^{-1}(0) = K$ and q is an embedding on $K' \setminus K$. Now define an embedding $j \colon K' \to Y \times Q$ as $j = p \times q$. Since $q^{-1}(0) = K$, the mapping j coincide with p on K which is inclusion on K. $\square$

Definition 2.4. We say that a filtration $F_0 \subset F_1 \subset \dots$ of multivalued mappings from X to Y *projectively contains* a filtration $G_0 \subset G_1 \subset \dots$ of multivalued mappings from X to $Y \times Q$ if for any point $x \in X$ and any n the set $\mathrm{pr}_Y \circ G_n(x)$ is contained in $F_n(x)$.

Theorem 2.5. *For a finite CW-complex L any fiberwise $[L]_c$-connected filtration of strongly l.s.c. multivalued mappings of paracompact space X to a topological space Y projectively contains stably $[L]$-connected filtration of compact-valued mappings.*

Proof. For a given fiberwise $[L]_c$-connected filtration $F_0 \subset F_1 \subset \dots$ of strongly l.s.c. multivalued mappings we construct stably $[L]$-connected filtration $G_0 \subset G_1 \subset \dots$ of compact-valued mappings $G_n \colon X \to Y \times Q^n$ as follows: successively for every $n \geq 0$ we construct a covering $\omega_n = \{W^n_\lambda\}_{\lambda \in \Lambda_n} \in \mathrm{cov}X$ and a family of subcompacta $\{K^n_\lambda\}_{\lambda \in \Lambda_n}$ of $Y \times Q^n$, and define the mapping G_n by the formula

$$G_n(x) = \cup\{K^n_\lambda \mid x \in W^n_\lambda\}.$$

First, we construct G_0, i.e. the covering ω_0 and the family $\{K^0_\lambda\}_{\lambda \in \Lambda_0}$. Since F_0 is strongly l.s.c., there exists a locally finite open covering $\omega_{-1} = \{W^{-1}_\lambda\}_{\lambda \in \Lambda_{-1}} \in \mathrm{cov}X$ and a family $\{M^{-1}_\lambda\}_{\lambda \in \Lambda_{-1}}$ of points in Y such that $W^{-1}_\lambda \times M^{-1}_\lambda \subset \Gamma_{F_0}$ for any $\lambda \in \Lambda_{-1}$. Denote by H_0 a multivalued mapping taking a point $x \in X$ to the set $H_0(x) = \cup\{M^{-1}_\lambda \mid x \in W^{-1}_\lambda\}$. Note that $H_0(x)$ is contained in $F_0(x)$ and consists of finitely many points. By Lemma 2.3 for any $x \in X$ there exists a compactum $\widehat{H}_0(x) \subset F_1(x) \times Q$ of extension dimension $\text{e-dim}\widehat{H}_0(x) \leq [L]$ such that the pair $H_0(x) \subset \widehat{H}_0(x)$ is $[L]$-connected. Since F_1 is strongly l.s.c., any point $x \in X$ has a neighborhood $\mathcal{O}_0(x)$ such that the product $\mathcal{O}_0(x) \times \widehat{H}_0(x)$ is contained in $\Gamma_{F_1} \times Q$. Since X is paracompact, we can choose neighborhoods $\mathcal{O}_0(x)$ in such a way that the covering $\mathcal{O}_0 = \{\mathcal{O}_0(x)\}_{x \in X}$ strongly star refines ω_{-1}. Let $\omega_0 = \{W^0_\lambda\}_{\lambda \in \Lambda_0}$ be a locally finite open cover of X refining $\mathcal{O}_0$. For every $\lambda \in \Lambda_0$ we fix a point x_λ such that $W^0_\lambda \subset \mathcal{O}_0(x_\lambda)$ and put $M^0_\lambda = \widehat{H}_0(x_\lambda)$. For every $\lambda \in \Lambda_0$ we fix $\alpha(\lambda) \in \Lambda_{-1}$ such that $\mathrm{St}(W^0_\lambda, \mathcal{O}_0) \subset W^{-1}_{\alpha(\lambda)}$ and put $K^0_\lambda = M^{-1}_{\alpha(\lambda)}$.

Inductive step of our construction is similar to the first step. Suppose that a covering $\omega_{n-1} = \{W^{n-1}_\lambda\}_{\lambda \in \Lambda_{n-1}} \in \mathrm{cov}X$ and a family $\{M^{n-1}_\lambda\}_{\lambda \in \Lambda_{n-1}}$ of compacta in $Y \times Q^{n-1}$ are already constructed such that $\text{e-dim}M^{n-1}_\lambda \leq [L]$ and the product $W^{n-1}_\lambda \times M^{n-1}_\lambda$ is contained in $\Gamma_{F_n} \times Q^n$ for any $\lambda \in \Lambda_{n-1}$. Denote by H_n a multivalued mapping taking a point $x \in X$ to the compactum $H_n(x) = \cup\{M^{n-1}_\lambda \mid x \in W^{n-1}_\lambda\}$. Note that $H_n(x)$ is contained in $F_n(x) \times Q^n$ and has extension dimension $\text{e-dim}H_n(x) \leq [L]$. By Lemma 2.3 for any $x \in X$ there exists a compactum $\widehat{H}_n(x) \subset F_{n+1}(x) \times Q^{n+1}$ of extension dimension $\text{e-dim}\widehat{H}_n(x) \leq [L]$ such that the pair $H_n(x) \subset \widehat{H}_n(x)$ is $[L]$-connected. Since F_{n+1} is strongly l.s.c., any point $x \in X$ has a neighborhood $\mathcal{O}_n(x)$ such that the product $\mathcal{O}_n(x) \times \widehat{H}_n(x)$ is contained in $\Gamma_{F_{n+1}} \times Q^{n+1}$. Since X is paracompact, we can choose neighborhoods $\mathcal{O}_n(x)$ in such a way that the covering $\mathcal{O}_n = \{\mathcal{O}_n(x)\}_{x \in X}$ strongly star refines ω_{n-1}. Let $\omega_n = \{W^n_\lambda\}_{\lambda \in \Lambda_n}$ be a locally finite open cover of X refining $\mathcal{O}_n$. For every $\lambda \in \Lambda_n$ we fix a point x_λ such that $W^n_\lambda \subset \mathcal{O}_n(x_\lambda)$ and put $M^n_\lambda = \widehat{H}_n(x_\lambda)$. For every $\lambda \in \Lambda_n$ we fix $\alpha(\lambda) \in \Lambda_{n-1}$ such that $\mathrm{St}(W^n_\lambda, \mathcal{O}_n) \subset W^{n-1}_{\alpha(\lambda)}$ and put $K^n_\lambda = M^{n-1}_{\alpha(\lambda)}$.

To show that the pair $G_{n-1} \subset G_n$ is stably $[L]$-connected, we prove that the pair $G_{n-1}(W^n_\lambda) \subset \cap\{G_n(x) \mid x \in W^n_\lambda\}$ is $[L]$-connected for any $W^n_\lambda \in \omega_n$. By the construction of G_n, the set K^n_λ is contained in $\cap\{G_n(x) \mid x \in W^n_\lambda\}$. We know that the pair $H_{n-1}(x_{\alpha(\lambda)}) \subset \widehat{H}_{n-1}(x_{\alpha(\lambda)}) = M^{n-1}_{\alpha(\lambda)} = K^n_\lambda$ is $[L]$-connected. Therefore it is enough to show the following inclusion:

$$G_{n-1}(W^n_\lambda) = \\ \bigcup\{K^{n-1}_\beta \mid W^n_\lambda \cap W^{n-1}_\beta \neq \emptyset\} \subset \cup\{M^{n-2}_\nu \mid x_{\alpha(\lambda)} \in W^{n-2}_\nu\} = H_{n-1}(x_{\alpha(\lambda)})$$

which follows from the fact that $W^n_\lambda \cap W^{n-1}_\beta \neq \emptyset$ implies $x_{\alpha(\lambda)} \in W^{n-2}_{\alpha(\beta)}$ (note that $M^{n-2}_{\alpha(\beta)} = K^{n-1}_\beta$). By the choice of $\alpha(\lambda)$ we have $W^n_\lambda \subset \mathcal{O}_{n-1}(x_{\alpha(\lambda)})$. Then $W^n_\lambda \cap W^{n-1}_\beta \neq \emptyset$ implies $\mathcal{O}_{n-1}(x_{\alpha(\lambda)}) \cap W^{n-1}_\beta \neq \emptyset$ and $x_{\alpha(\lambda)} \in \mathcal{O}_{n-1}(x_{\alpha(\lambda)}) \subset \mathrm{St}(W^{n-1}_\beta, \mathcal{O}_{n-1}) \subset W^{n-2}_{\alpha(\beta)}$. □

Definition 2.6. For a space Z a pair of spaces $V \subset U$ is said to be *Z-connected* if for every closed subspace $A \subset Z$ any mapping of A into V can be extended to a mapping of Z into U.

Definition 2.7. A pair $F \subset H$ of multivalued mappings from X to Y is called *stably Z-connected* if every point $x \in X$ has a neighborhood O_x such that the pair $F(O_x) \subset \cap_{z\in O_x} H(z)$ is Z-connected.

We say that the pair $F \subset H$ is called *stably Z-connected with respect to a covering* $\omega \in \mathrm{cov}X$, if for any $W \in \omega$ the pair $F(W) \subset \cap_{x\in W} H(x)$ is Z-connected.

A filtration $\{F_i\}$ of multivalued mappings is called *stably Z-connected* if every pair $F_i \subset F_{i+1}$ is stably Z-connected.

Theorem 2.8. *Let $F\colon X \to Y$ be a multivalued mapping of paracompact C-space X to a topological space Y. If F admits infinite stably X-connected filtration of multivalued mappings, then F has a singlevalued continuous selection.*

Proof. Let $\{F_i\}_{i=-1}^\infty$ be the given filtration of F. Let $\{\omega_i\}_{i=-1}^\infty$ be a sequence of coverings of X such that ω_{i+1} refines ω_i and the pair $F_i \subset F_{i+1}$ is stably X-connected with respect to the covering ω_i. Since X is paracompact C-space, there exists a locally finite closed cover Σ of X of the form $\Sigma = \cup_{i=0}^\infty \sigma_i$ such that σ_i is discrete collection refining ω_i. Define $\Sigma_n = \cup_{i=0}^n \sigma_i$. We will construct a continuous selection f of F extending it successively over the sets Σ_n.

First, we construct $f_0\colon \Sigma_0 \to Y$. We define f_0 separately on every element s of the discrete collection σ_0: take a point $p \in F_{-1}(s)$ and put $f_0(s) = p$. Since the set s refines ω_0, then $p \in F_0(x)$ for any $x \in s$ and therefore f_0 is a selection of $F_0|_{\Sigma_0}$.

Suppose that we already constructed f_n — a continuous selection of $F_n|_{\Sigma_n}$. Let us define f_{n+1} on arbitrary element Z of discrete collection σ_{n+1}. Since Σ is locally finite, the set $A = Z \cap \Sigma_n$ is closed in X. Since f_n is a selection of F_n, then $f_n(A)$ is contained in $F_n(Z)$. Since the pair $F_n(Z) \subset \cap_{x\in Z} F_{n+1}(x)$ is

X-connected, we can extend $f_n|_A$ to a mapping $f'_n\colon Z \to \cap_{x\in Z} F_{n+1}(x)$. Clearly, f'_n is a selection of $F_{n+1}|_Z$. We define f_{n+1} on the set Z as f'_n.

Finally, we define f to be equal to f_n on the set Σ_n. □

Theorem 2.9. *Let L be a finite CW-complex and $F\colon X \to Y$ be a multivalued mapping of paracompact C-space X of extension dimension* e-dim$X \le [L]$ *to a topological space Y. If F admits infinite fiberwise $[L]_c$-connected filtration of strongly l.s.c. multivalued mappings, then F has a singlevalued continuous selection.*

Proof. By Theorem 2.5, the mapping $F'\colon X \to Y \times Q$ defined as $F'(x) = F(x)\times Q$ contains a stably $[L]$-connected filtration of multivalued mappings. By Theorem 2.8 F' has a singlevalued continuous selection f'. Then the mapping $f = \mathrm{pr}_Y \circ f'$ is a singlevalued continuous selection of F. □

3. Hurewicz theorem

The proof of the following theorem is similar to the proof of Theorem 2.4 from [3].

Theorem 3.1. *Let $f\colon X \to Y$ be a closed mapping of k-space X onto paracompact C-space Y. Suppose that* e-dim$Y \le [M]$ *for a finite CW-complex M. If for every point $y \in Y$ and for every compactum Z with* e-dim$Z \le [M]$ *we have* e-dim$(f^{-1}(y)\times Z) \le [L]$ *for some CW-complex L, then* e-dim$X \le [L]$.

Proof. Suppose $A \subset X$ is closed and $g\colon A \to L$ is a map. We are going to find a continuous extension $\widetilde{g}\colon X \to L$ of g. Let K be the cone over L with a vertex v. We denote by $C(X,K)$ the space of all continuous maps from X to K equipped with the compact-open topology. We define a multivalued map $F\colon Y \to C(X,K)$ as follows:

$$F(y) = \{h \in C(X,K) \mid h(f^{-1}(y)) \subset K \setminus \{v\} \text{ and } h|_A = g\}.$$

Claim. F admits continuous singlevalued selection.

If $\varphi\colon Y \to C(X,K)$ is a continuous selection for F, then the mapping $h\colon X \to K$ defined by $h(x) = \varphi(f(x))(x)$ is continuous on every compact subset of X and because X is a k-space, h is continuous. Since $\varphi(f(x)) \in F(f(x))$ for every $x \in X$, we have $h(X) \subset K \setminus \{v\}$. Now if $\pi\colon K\setminus\{v\} \to L$ denotes the natural retraction, then $\widetilde{g} = \pi \circ h\colon X \to L$ is the desired continuous extension of h.

Proof of the claim. We are going to apply Theorem 2.9 to infinite filtration $F \subset F \subset F \subset \dots$. To do this, we have to show that F is strongly l.s.c. and that the pair $F(y) \subset F(y)$ is $[M]_c$-connected for every point $y \in Y$.

First, we show that F is strongly l.s.c. Let $y_0 \in Y$ and $P \subset F(y_0)$ be compact. We have to find a neighborhood V of y_0 in Y such that $P \subset F(y)$ for every $y \in V$. For every $x \in X$ define a subset $P(x) = \{h(x) \mid h \in P\}$ of K. Since $P \subset C(X,K)$ is compact and X is a k-space, by the Ascoli theorem, each

$P(x)$ is compact and P is evenly continuous. This easily implies that the set $W = \{x \in X \mid P(x) \subset K \setminus \{v\}\}$ is open in X and, obviously, $f^{-1}(y_0) \subset W$. Since f is closed, there exists a neighborhood V of y_0 in Y with $f^{-1}(V) \subset W$. Then, according to the choice of W and the definition of F, we have $P \subset F(y)$ for every $y \in V$.

Fix an arbitrary point $y \in Y$. Let us prove that the pair $F(y) \subset F(y)$ is $[M]_c$-connected. Consider a pair of compacta $B \subset Z$ where e-dim$Z \leq [M]$ and a mapping $\varphi\colon B \to F(y)$. Since $B \times X$ is a k-space (as a product of a compact space and a k-space), the map $\psi\colon B \times X \to K$ defined as $\psi(b,x) = \varphi(b)(x)$ is continuous. Extend ψ to a set $Z \times A$ letting $\psi(z,a) = g(a)$. Clearly, ψ takes the set $Z \times f^{-1}(y) \cap (Z \times A \cup B \times X)$ into $K \setminus \{v\} \cong L \times [0,1)$. Since e-dim$(Z \times f^{-1}(y)) \leq [L]$, we can extend ψ over the set $Z \times f^{-1}(y)$ to take it into $K \setminus \{v\}$. Finally extend ψ over $Z \times X$ as a mapping into AE-space K. Now define an extension $\widetilde{\varphi}\colon Z \to F(y)$ of the mapping φ by the formula $\widetilde{\varphi}(z)(x) = \psi(z,x)$. □

Corollary 3.2 (cf. Theorem 2.25 from [6])**.** *Let $f\colon X \to Y$ be a mapping of finite-dimensional compacta where* e-dim$Y = [M]$ *for finite CW-complex M. If for some CW-complex L we have* e-dim$(f^{-1}(y) \times Y) \leq [L]$ *for every point $y \in Y$, then* e-dim$X \leq [L]$.

Proof. By Theorem 6.3 from [5] for any compactum Z with e-dim$Z \leq$ e-dim$Y$ we have e-dim$(f^{-1}(y) \times Z) \leq [L]$. Thus, we can apply Theorem 3.1 □

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